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Momentum Exchange: Feedback Control of Flexible Spacecraft Maneuvers and Vibration

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The momentum exchange controller in which the time rate of change of the flexible momentum relative to the rigid-body motion is used as a part of the feedback control law for maneuvers and vibration suppression of flexible systems is introduced. This control concept is applied to a model of a rigid hub (base) with a cantilevered flexible appendage undergoing a single-axis maneuver. The feedback control on the hub includes the rigid-body motion and the time rate of change of that part of the flexible momentum resulting from flexible vibration. The lower and upper bounds of the control law for system Lyapunov stability are obtained, and the relationship of the control law with the energy Lyapunov test function is established. With the presence of this feedback control, an additional independent flexible control system acting on the flexible parts can be designed for further vibration suppression. This hybrid control system can be applied to both stationkeeping and large-angle maneuvers. Both analytical and numerical results are presented to show the theoretical and practical merit of this approach.

I. Introduction

THE control of flexible systems usually requires the controller to provide the control effort for maneuvering or targeting of the flexible systems/subsystems (such as antennas and telerobots) with the simultaneous vibration suppression. For instance, one of the major problems with robotic manipulator systems is the assembly time lost while waiting for the vibration suppression of the end of the manipulator arm.

The mathematical modeling of flexible systems involves the coupling of finite and infinite ordered mathematical subsystems—the rigid-body motions described by ordinary differential equations and the flexible motions described by partial differential equations. The control objective is to maneuver the rigid body and suppress the flexible vibration simultaneously for targeting, or to suppress both motions for stationkeeping. Most previous approaches to this problem resorted to the discretization of the continuous systems into finite-dimensional systems as the first stage of the process for control law design. It is commonly recognized that the discretization procedure inevitably involves modeling errors and control spillover issues. Also, since the frequencies of the flexible vibration are usually much higher than those of rigid-body motions, the coupling of these two motions will result in the stiffening of the system differential equations. As a consequence, the numerical solutions to the traditionally optimal approaches, such as minimizing a quadratic cost functional or minimum-time bang-bang control, are usually difficult to solve because of the inherent nonlinearity and stiffness of the resulting differential equations. In addition, the algorithms and control laws are usually sensitive to modeling errors and may lack robustness.

To suppress the flexible vibrations, certain quantities that contain the information about the flexible motion of the distributed systems are needed for the feedback controller. What are these quantities for continuous mechanical systems? From the viewpoint of dynamics, the rigid-body motion and the flexible motion interact with each other by means of momentum exchange during maneuvering; hence, the part of the flexible momentum resulting from the flexible motion is expected to contain the information about the flexible vibration and can be used as a part of the feedback law. The main purpose of the present paper is to introduce the so-called momentum exchange control concept, which is illustrated by the application to a particular model of a rigid hub with a flexible appendage beam undergoing a single-axis maneuver. The proposed control acting on the hub includes the feedback of the rigid-body motion and the time rate of change of the flexible momentum of the beam (combination of the flexible modes). We will strictly prove the stability of this control law by using a Lyapunov test function, and the lower and upper bounds of the control law for system Lyapunov stability will be obtained. Many interesting and important implications of this control concept will be demonstrated with the development.

Concerning the related research, attention should be directed to recent impressive work by Junkins et al.,[1,2] who considered the maneuver and vibration suppression of a rigid hub with flexible appendage beams, and their proposed control law involved the feedback of the combination of the root shear and bending moment of the beam, which theoretically was equivalent to the time rate of change of the angular momentum of the beam, i.e., the combination of the rigid momentum plus the flexible momentum. The idea was innovative and the resulting control was easily implemented from the viewpoint of measurement; however, from the viewpoint of the present investigation, it appears that the inclusion of the rigid momentum of the beam into the feedback control law is redundant and may corrupt the information about the flexible vibration with possible degradation of the system performance. This argument is further suported by the fact that there is no finite upper bound limit of the control gains required for system stability in Ref. 1. The corresponding version of the development of Junkins' control law is given in the Appendix for the purpose of comparison.

Another aim of the present paper is to design an additional independent flexible feedback control system for further vibration control. Independent flexible feedback control means that the feedback of this control system does not depend on either the open-loop rigid-body maneuver strategy or on the precalculated reference flexible motion; the feedback depends on only the instantaneous flexible displacements and velocities. In the present paper, it is proposed to further suppress the vibrations by the natural modal control.

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Therefore, together with the presence of the control acting on the rigid hub, an additional natural modal control acting on the flexible beam for further vibration suppression will also be employed. This flexible control system is said to be independent in the aforementioned sense, and it will be shown that the system stability is guaranteed as long as the flexible control forces are energy-dissipating. This proposed hybrid control system will be shown to be effective for vibration suppression during both stationkeeping and large-angle maneuvering.

In summary, the objectives of this paper are to 1) present a kind of momentum exchange feedback control concept and prove the system stability by the Lyapunov direct method; 2) design an additional independent flexible control system for further vibration control; and 3) apply this hybrid control system to the large-angle maneuver problem of the flexible system.

II. Momentum Exchange Feedback Control

A rigid hub with a cantilevered flexible appendage (Fig. 1) is considered. The appendage is considered to be a uniform flexible beam, and the shear deformation and distributed rotary inertia are neglected. The beam is assumed to undergo small deformations so that linear elastic theory can be applied. The axial deformation is also neglected. Motion is restricted to the horizontal plane, and, at this stage of development, only a control torque acting on the hub is considered.

With reference to Fig. 1, the position vector of a differential element dm in the undeformed body-fixed frame can be expressed as

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} \tag{1}$$

The inertial velocity of dm can be obtained as

$$\dot{\boldsymbol{r}} = -y\dot{\theta}\boldsymbol{i} + (x\dot{\theta} + \dot{y})\boldsymbol{j} \tag{2}$$

Therefore, the total inertial angular momentum with respect to point O can be written as

$$\boldsymbol{H} = \left[(I_h + I_b)\dot{\theta} + \int_{l_0}^{l} \rho(x\dot{y} + y^2\dot{\theta})\,\mathrm{d}x\right]\boldsymbol{k} \tag{3}$$

where I_h is the moment of inertia of the rigid hub with respect to point O, and

$$I_b = \int_{l_0}^{l} \rho x^2\,\mathrm{d}x$$

is the moment of inertia of the undeformed beam. The part of the angular momentum associated with the beam's flexible motion is denoted

$$H_f = \int_{l_0}^{l} \rho(x\dot{y} + y^2\dot{\theta})\,\mathrm{d}x$$

and is called the flexible momentum of the beam.

After applying the Euler-Newtonian equation of motion, one can obtain the rotational motion equation for the rigid-body motion as

$$(I_h + I_b)\ddot{\theta} + \frac{\mathrm{d}}{\mathrm{d}t}H_f = u(t) \tag{4}$$

where $u(t)$ is the control torque acting on the hub.

The partial differential equation governing the elastic vibration of the beam can be written as

$$\rho\left(\frac{\partial^2 y}{\partial t^2} + x\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right) + EI\frac{\partial^4 y}{\partial x^4} = 0 + \mathrm{HOT} \tag{5}$$

where HOT denotes higher-order terms that include other known effects [i.e., rotational stiffening ($y\dot{\theta}^2$) and shear de-

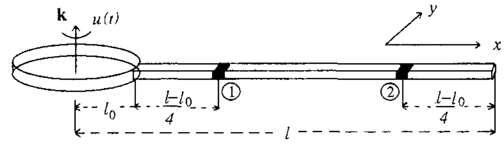

Fig. 1 System configuration and lumped-mass model.

formation). The fundamental development given here does not consider these higher-order terms.

The boundary conditions on Eq. (5) are

$$\text{at} \quad x = l_0, \quad y(t, l_0) = \left.\frac{\partial y}{\partial x}\right|_{x=l_0} = 0$$

$$\text{at} \quad x = l, \quad \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=l} = \left.\frac{\partial^3 y}{\partial x^3}\right|_{x=l} = 0 \tag{6}$$

A feedback control law for stationkeeping is introduced as

$$u(t) = -k_1\theta - k_2\dot{\theta} - \gamma\frac{\mathrm{d}}{\mathrm{d}t}H_f \tag{7}$$

Notice that, although the flexible momentum time rate

$$\frac{\mathrm{d}}{\mathrm{d}t}H_f$$

is not directly measureable, it can be constructed by combining the strain gauge measurement of the root shear, the root bending moment, and the accelerometer measurement of the angular acceleration ($\ddot{\theta}$) (see the Appendix for details).

Also, notice that the form of this feedback control is different from that presented in Junkins et al.,[1,2] where the control involved the feedback of both the root shear and bending moment of the beam, and the combination of these moments was equal to the time rate of change of the angular momentum of the beam; i.e., the combination of the rigid momentum plus the flexible momentum.

The Lyapunov test function ("error energy") is taken as the following form:

$$V = \frac{1}{2}a_1(I_h + I_b)\dot{\theta}^2 + \frac{1}{2}a_2\theta^2 + \frac{1}{2}a_3\left[\int_{l_0}^{l}\rho\left(\frac{\partial y}{\partial t} + x\dot{\theta}\right)^2\mathrm{d}x + \int_{l_0}^{l}EI\left(\frac{\partial^2 y}{\partial x^2}\right)^2\mathrm{d}x\right] \tag{8}$$

which is a modification of the form used in Ref. 1. However, instead of $\frac{1}{2}a_1 I_h\dot{\theta}^2$ in Ref. 1, we use $\frac{1}{2}a_1(I_h + I_b)\dot{\theta}^2$, the kinetic energy of the rigid-body motion of the whole system, in the first term. This effectively separates the rigid beam energy (and combines it with the hub energy) from the contributions due to beam flexure. The second term, $a_2\theta^2$, the "torsional spring energy," is added to make the final state $(\theta, \dot{\theta}, y, \partial y/\partial t) = (0, 0, 0, 0)$ be the global minimum of V. The third term is the kinetic and potential energy of the beam. The weighting coefficients a_i are included to allow relative emphasis on the three contributors to the error energy of the system.

It is obvious by inspection that imposing $a_i > 0$ guarantees that $V \geq 0$ and that the global minimum of $V = 0$ occurs only at the zero state. Differentiation, substitution of the equations of motion (5) and boundary conditions (6), and some calculus manipulations lead to

$$\dot{V} = a_1(I_h + I_b)\dot{\theta}\ddot{\theta} + a_2\theta\dot{\theta} + a_3\dot{\theta}EI\left(l_0\left.\frac{\partial^3 y}{\partial x^3}\right|_{x=l_0} - \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=l_0}\right) \tag{9}$$

Furthermore, for the rigid hub we have

$$I_h\ddot{\theta} = u(t) - EI\left(l_0\frac{\partial^3 y}{\partial x^3}\bigg|_{x=l_0} - \frac{\partial^2 y}{\partial x^2}\right)\bigg|_{x=l_0} \tag{10}$$

After substituting Eqs. (4), (7), and (10) into Eq. (9), one can obtain

$$\dot{V} = \left\{\left[a_2 - k_1\left(a_1 + \frac{a_3 I_b}{I_h + I_b}\right)\right]\theta - k_2\left(a_1 + \frac{a_3 I_b}{I_h + I_b}\right)\dot{\theta} - \left[\left(a_1 - \frac{a_3 I_h}{I_h + I_b}\right) + \gamma\left(a_1 + \frac{a_3 I_b}{I_h + I_b}\right)\right]\frac{\mathrm{d}}{\mathrm{d}t}H_f\right\}\dot{\theta} \tag{11}$$

To meet the requirement that $\dot{V} \le 0$ to guarantee stability, two of the coefficients in Eq. (11) are set to zero:

$$a_2 - k_1\left(a_1 + \frac{a_3 I_b}{I_h + I_b}\right) = 0 \tag{12}$$

$$\left(a_1 - \frac{a_3 I_h}{I_h + I_b}\right) + \gamma\left(a_1 + \frac{a_3 I_b}{I_h + I_b}\right) = 0 \tag{13}$$

In this case $\dot{V}$ becomes

$$\dot{V} = -k_2\left(a_1 + \frac{a_3 I_b}{I_h + I_b}\right)\dot{\theta}^2 \le 0 \qquad (\text{if} \quad k_2 \ge 0) \tag{14}$$

Equations (12–14) lead to the following control gain requirements:

$$k_2 \ge 0 \tag{15}$$

$$k_1 = \frac{a_2(I_h + I_b)}{a_3 I_b + a_1(I_h + I_b)} \ge 0 \tag{16}$$

$$\gamma = \frac{a_3 I_h - a_1(I_h + I_b)}{a_3 I_b + a_1(I_h + I_b)} \tag{17}$$

Since $a_i \ge 0$, it can be easily proven that

$$-1 \le \gamma \le \frac{I_h}{I_b} \tag{18}$$

and $\gamma = -1$ when $a_3 = 0$; $\gamma = I_h/I_b$ when $a_1 = 0$.

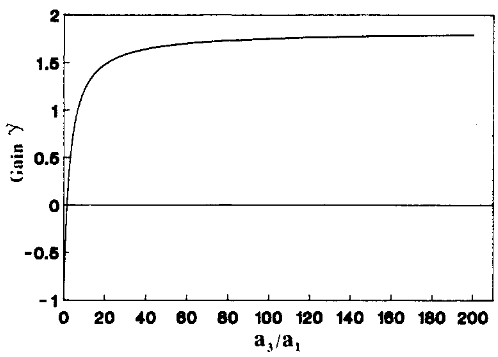

Fig. 2 Variable γ as a function of a_3/a_1.

Table 1 Spacecraft dimensions, appendage material

Radius of the rigid central body, l_0	1.00 m
Length of the appendage, l	20.00 m
Appendage material stiffness, EI	1500.00 N-m^2
Appendage material density, ρ	0.04096 kg/m
Mass of the rigid hub	400.00 kg
Total rotational inertia, $I_h + I_b$	309.19 kg/m^2
Inertia ratio, I_h/I_b	1.83

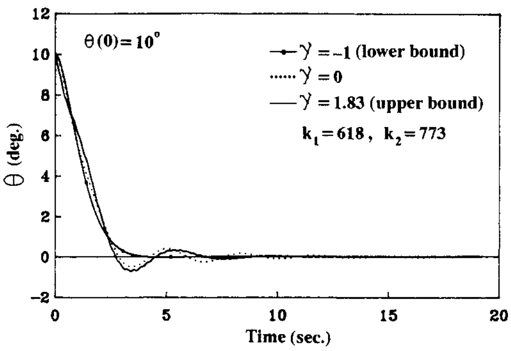

Fig. 3 Rigid-body rotation responses in stationkeeping.

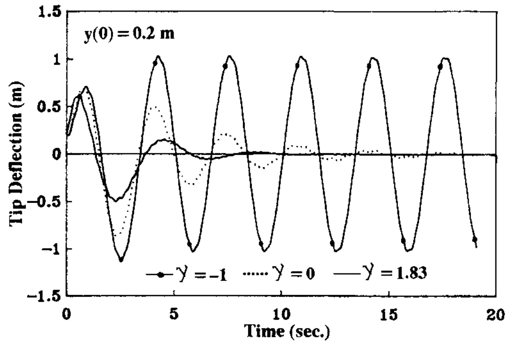

Fig. 4 Flexible vibrations in stationkeeping.

Thus, a finite lower and upper bound restricts the size of the control gain γ. It should be noted that there was no such finite upper bound limit in Refs. 1 and 2. Figure 2 shows γ as a function of a_3/a_1 (all numerical data are from Table 1).

Now attention is directed to the significant physical meaning of this feedback control law. According to Eq. (8), a_3/a_1 corresponds to the relative contribution of the flexible motion and the rigid-body motion to the system error energy. When $a_3 = 0$, which corresponds to $\gamma = -1$, the error energy Lyapunov function does not contain the flexible energy term. In other words, the emphasis is totally placed on the control of the rigid-body motion. For the case where $\gamma = -1$, it can be seen from Eqs. (4) and (7) that the effect of the flexible momentum H_f is removed from the closed-loop rigid-body motion equation. This means that the controller completely compensates for the flexible motion, and the rigid-body motion is not disturbed by the flexible vibration. This result coincides with the physical intuition. The opposite explanation is true for the case when $a_1 = 0$.

For numerical simulation, a simple two-lumped mass beam model is used to discretize the continuous beam (see also Fig. 1). However, the theoretical development and the validation of the results do not depend on the discretization procedure. Figures 3 and 4 show the transient response of the rigid-body motion and the flexible motion, respectively. The three cases considered are $\gamma = -1$, maximum emphasis on the rigid-body motion (complete compensation for the flexible motion); $\gamma = 0$, no feedback of the flexible momentum; and $\gamma = I_h/I_b = 1.83$, maximum emphasis on the flexible motion. It can be seen that, with the increase of γ (larger relative emphasis on the suppression of the flexible motion), the dissipation of the flexible vibrations is better, and the rigid-body motion is subjected to more disturbance, but the disturbance is not large enough to degrade the rigid-body motion seriously. Therefore, in practical applications γ can be selected according to the requirement for the rigid-body motion and the flexible vibration suppression and is a kind of tradeoff. The selection of the control gain can be further enhanced by combining some optimum criteria.[2] Figure 5 shows the control torque requirement for the case when $\gamma = 1.83$.

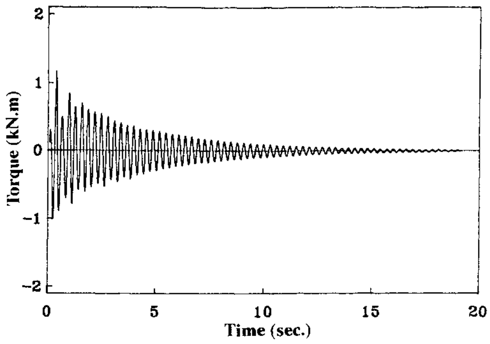

Fig. 5 Typical control torque profile ($\gamma = 1.83$).

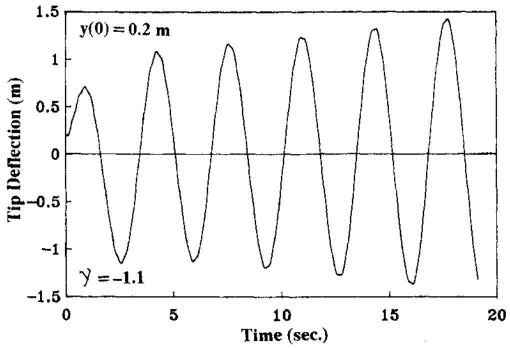

Fig. 6 Divergent flexible motion.

It should also be noted that bounded restrictions for the gain γ are only sufficient conditions for system stability; however, from the present simulation it is seen that these restrictions are close to the necessary conditions for system stability, especially the lower bound condition. Figures 6 and 7 show that when γ exceeds the lower bound the flexible motion becomes unstable and vice versa, when γ exceeds the upper bound the rigid-body motion diverges.

The system performance will generally depend on the ratio I_h/I_b. Figures 8 and 9 show the system responses corresponding to three different ratios. It is seen that, if this ratio increases, the flexible vibration will be usually easier to be suppressed and the rigid-body motion will also decay more quickly, with a little bit larger overshoot at the beginning. From the simulation results it is also seen that this control law is robust in the presence of other unmodeled factors and modeling errors, and the robustness will degrade when the control gain γ approaches its boundaries.

Finally, it should be pointed out that the current control law [Eq. (7)] could be derived from Eq. (8), a modification of the energy weighting in the Lyapunov function in Ref. 1, as the simplest stabilizing control law. The current form of the Lyapunov function separates the rigid beam energy from the contribution due to the beam flexure and combines it with the hub energy. The consequence of this small change is significant, and some additional qualitative insight has been shown. However, it should be emphasized that the current control law is independently introduced. The Lyapunov function serves as a tool for us to gain insight into the stability characteristics of the closed-loop dynamics. The fundamental idea behind this approach is that the coupled subsystems (motions), such as the rigid-body motion and the flexible motion, are seen to interact with each other through the boundary forces or torques, which are associated with the time rate of change of the linear or angular momentum of the subsystems according to Newton's second law. Therefore, the time rate of change of that part of the momentum resulting from a particular subsystem (such as the flexible motion) must contain the information about the subsystem and can be used as a part of information feedback to control the subsystem. The physical explanation of the present controller is that the momentum of the flexible vibration (thus the energy of the flexible motion) is transferred to the rigid-body motion, which, in turn, is absorbed by the controller (momentum exchange). This momentum exchange control concept, therefore, is general and can be applied to other problems.[3]

III. Independent Flexible Control for Elastic Vibration

In Sec. II, it has been demonstrated that the feedback controller located on the rigid part of the spacecraft can perform the maneuver and simultaneous vibration suppression, and the system stability is guaranteed as long as the control law gains are within a certain range. However, it is of interest to further control and suppress the flexible vibration while the spacecraft is experiencing large angle rotations and large translations. To achieve this, a flexible modal control system acting on the flexible appendage is used.

Baruh and Silverberg[4] have proven that the natural modal control forces conserve the linear and angular momentum of the spacecraft and, therefore, do not appear on the right-hand

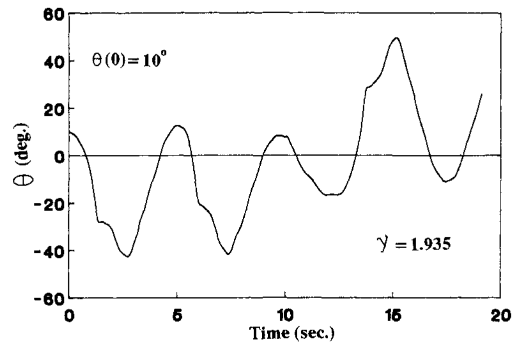

Fig. 7 Divergent rigid-body motion.

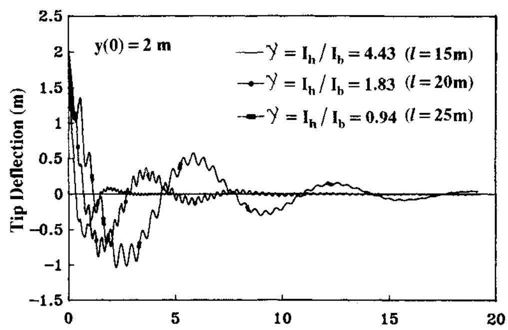

Fig. 8 Transient responses (flexible vibration) when I_h/I_b changes.

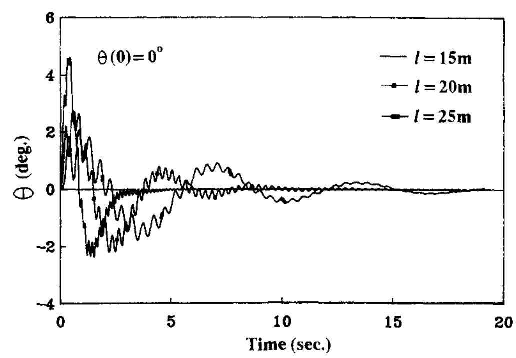

Fig. 9 Transient responses (rigid-body motion) when I_h/I_b changes.

side of the rigid-body equations. However, the rigid-body motion is still affected by the flexible motion via the kinematic and dynamic coupling, which is sometimes very important. This coupling was simply assumed to be small and was thus neglected in the discussion of Ref. 4. With the feedback control presented in Sec. II, this restrictive assumption on the coupling will be abandoned, and it will be proven that the system stability is guaranteed as long as the elastic modal control system is an energy-dissipating system. To prove this, it is assumed that u_e is the control on the flexible appendage, and u_e can be written as

$$u_e = \Sigma \phi_r f_r \tag{19}$$

where ϕ_r is the rth natural modal shape and f_r is the corresponding modal control force, which can be designed in the modal space using various methods, such as optimal linear quadratic regulator (LQR), pole assignment, or direct velocity feedback.

In this case, Eq. (5) becomes

$$\rho\left(\frac{\partial^2 y}{\partial t^2} + x\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right) + EI\frac{\partial^4 y}{\partial x^4} = u_e \tag{20}$$

The Lyapunov function is taken the same as Eq. (8). Using the orthogonality relation of the natural modal control (see the details in Ref. 4), and after similar manipulations, the derivative of V can be obtained as

$$\dot{V} = \text{right-hand side of Eq. (11)} + \int_{l_0}^{l} \rho \dot{y} u_e \,\mathrm{d}x \tag{21}$$

Therefore, if

$$\int_{l_0}^{l} \rho \dot{y} u_e \,\mathrm{d}x \le 0 \tag{22}$$

which means the control forces are energy-dissipating, then $\dot{V} \le 0$ and the system is stable.

The simulation results show that the addition of this elastic control system further suppresses the elastic vibration and improves the system responses (Figs. 10 and 11).

The flexible control system acting on the flexible beam, in fact, could be implemented by other control technologies, passive or active, such as the use of a distributed piezoelectric actuator. This hybrid control system (combines the momentum exchange and independent flexible control, a hierarchy control structure) provides some potential advantages. One of the advantages is that the central control system acting on the rigid part guarantees the system stability and the independent flexible control system further suppresses the vibrations. The effectiveness of this control approach will be further demonstrated in the problem of large angle maneuvers.

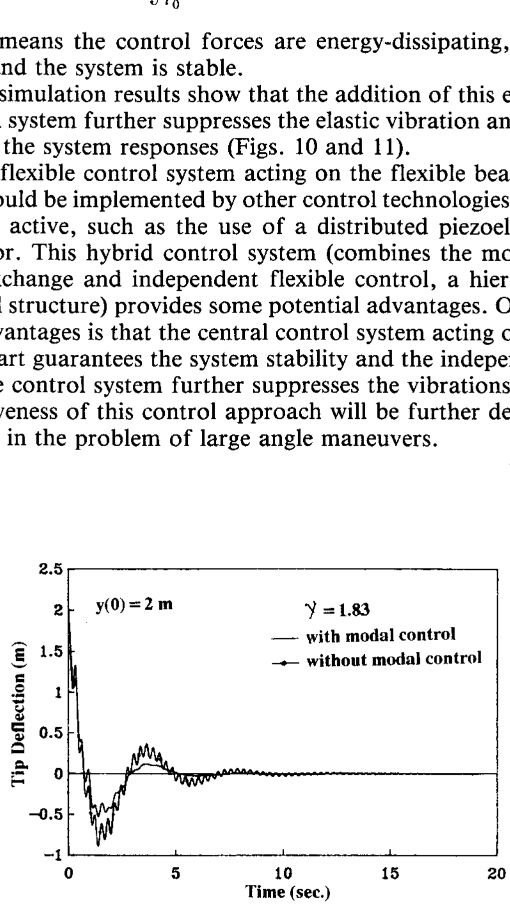

Fig. 10 Transient responses (flexible vibration) with and without the additional flexible modal control.

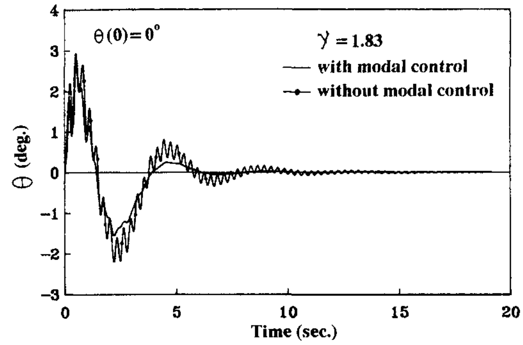

Fig. 11 Transient responses (rigid-body motion) with and without the additional flexible modal control.

IV. Large Angle Maneuvers

Motivated by Ref. 1, we now consider a tracking-type control more suitable for large angle maneuvers. A reference open-loop rigid-body maneuver is adopted, denoted by the subscripts as $\{\theta_r(t), \dot{\theta}_r(t), \ddot{\theta}_r(t); u_r(t)\}$, satisfying

$$(I_h + I_b)\ddot{\theta}_r = u_r(t) \tag{23}$$

and the prescribed boundary conditions. The variables without subscripts represent the actual solutions to the system Eqs. (4), (7), and (20).

The tracking control law takes the following form:

$$u(t) = u_r - k_1(\theta - \theta_r) - k_2(\dot{\theta} - \dot{\theta}_r) - \gamma \frac{\mathrm{d}}{\mathrm{d}t} H_f \tag{24}$$

The error energy Lyapunov function is modified as

$$V = \frac{1}{2} a_1 (I_h + I_b)(\dot{\theta} - \dot{\theta}_r)^2 + \frac{1}{2} a_2 (\theta - \theta_r)^2 + \frac{1}{2} a_3 \left\{ \int_{l_0}^{l} \rho \left[\frac{\partial y}{\partial t} + x(\dot{\theta} - \dot{\theta}_r) \right]^2 \mathrm{d}x + \int_{l_0}^{l} EI \left(\frac{\partial^2 y}{\partial x^2} \right)^2 \mathrm{d}x \right\} \tag{25}$$

The same manipulations as those in Sec. II lead to

$$\begin{aligned}\dot{V} = \Bigg\{ & \left[a_2 - k_1 \left(a_1 + \frac{a_3 I_b}{I_h + I_b} \right) \right] (\theta - \theta_r) \\ & - k_2 \left(a_1 + \frac{a_3 I_b}{I_h + I_b} \right) (\dot{\theta} - \dot{\theta}_r) - \left[\left(a_1 - \frac{a_3 I_h}{I_h + I_b} \right) \right. \\ & \left. + \gamma \left(a_1 + \frac{a_3 I_b}{I_h + I_b} \right) \right] \frac{\mathrm{d}}{\mathrm{d}t} H_f \Bigg\} (\dot{\theta} - \dot{\theta}_r) \\ & + a_3 \int_{l_0}^{l} \rho \dot{y} u_e \,\mathrm{d}x - a_3 \int_{l_0}^{l} \rho x \dot{y} \,\mathrm{d}x \ddot{\theta}_r \end{aligned} \tag{26}$$

Again, the tracking control laws are subjected to the same requirements as those in Eqs. (15–17). In this case,

$$\dot{V} = -k_2 \left(a_1 + \frac{a_3 I_b}{I_h + I_b} \right) (\dot{\theta} - \dot{\theta}_r)^2 + a_3 \int_{l_0}^{l} \rho \dot{y} u_e \,\mathrm{d}x - a_3 \int_{l_0}^{l} \rho x \dot{y} \,\mathrm{d}x \ddot{\theta}_r \tag{27}$$

in which the first term is seminegative definite; the second term is negative definite if the flexible control force is energy-dissipating; and the sign of the third term is generally varying. Therefore, as a whole, we cannot say the $\dot{V}$ is negative. In effect, the system has continuous inputs and the flexible vibra-

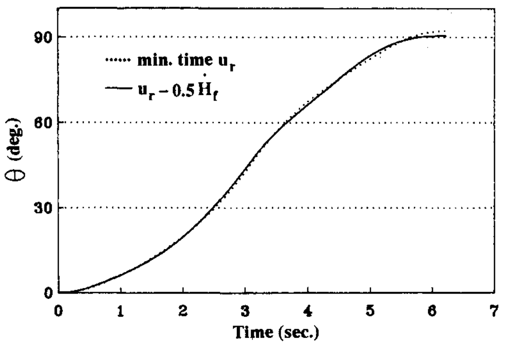

Fig. 12 Rigid-body rotation: 90-deg angular maneuver.

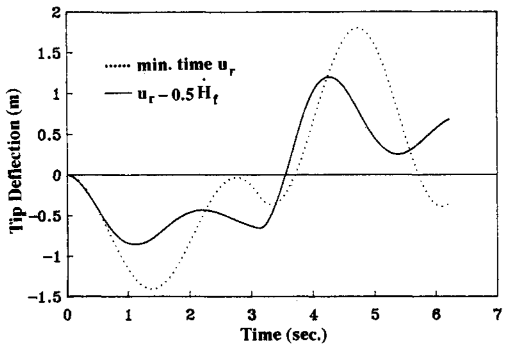

Fig. 13 Flexible vibration: 90-deg angular maneuver.

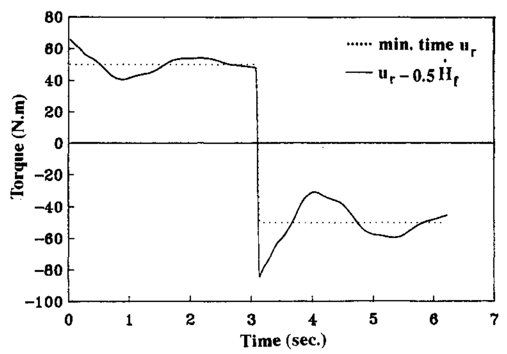

Fig. 14 Control torque profile: 90-deg angular maneuver.

tion will be excited during large angle maneuvers. Alternatively, we note that Ref. 1 developed a method for establishing a priori bounds on the region where $\dot{V}$ is negative definite, and it is anticipated that similar methods could be used to extend the present results. It is hoped that with this hybrid control the flexible vibration will be suppressed and that the bounded-input/bounded-output viewpoint of stability can be established.

It is not the purpose of this paper to discuss the open-loop maneuver strategy. A review of this subject can be found in Ref. 5. For the purpose of demonstration, a minimum-time maneuver strategy that is a bang-bang law is considered. It is recognized that a minimum-time maneuver bang-bang law will excite the flexible vibrations excessively, and it is intentionally used here to show the effectiveness of the proposed hybrid feedback control.

For quick large angle maneuvers, k_1 and k_2 are usually taken to be zero. Notice that in this case the remaining feedback term,

$$-\gamma \frac{\mathrm{d}}{\mathrm{d}t} H_f$$

does not depend on the open-loop maneuver strategy and the precalculated corresponding reference flexible motion. The results of a 90-deg maneuver are shown in Figs. 12–14. It is apparent from Fig. 13 that the (tip) deflection of the outermost lumped mass is reduced by almost half when the flexible momentum is included as a feedback term in the tracking control law.

V. Concluding Remarks

A novel, so-called momentum exchange feedback control concept is introduced. Instead of the feedback of the combination of the root shear and bending moment given in Ref. 1, the time rate of change of the beam flexible momentum as a feedback term is used; this results in a separation between the unwarranted beam rigid-body motion and the contribution due to the beam flexure. By offering a modification of the energy weighting in the Lyapunov function of Eq. (8) in Ref. 1, some significant qualitative insight into the current control law has been obtained. It should be pointed out that the current control law could also be derived from the modification of the Lyapunov function as in Ref. 1. (However, the current control law is independently introduced in the present paper.) This fact illustrates that, if a Lyapunov function is properly selected, a control law based on physical intuition can be obtained directly from the Lyapunov function approach.

The proposed hybrid control system (combining the momentum exchange controller acting on the rigid part and the independent flexible control system on the flexible part) has been shown to be effective in suppressing the flexible vibrations during both stationkeeping and large angle maneuvering. The significant advantages of this control approach include the following: 1) No discretization procedure for distributed parameter systems is necessary in the control design and, therefore, control/observation spillover issues are avoided; 2) only a small number of output is needed to implement the control; and 3) additional independent flexible control systems can be designed to further enhance the system performance. This control scheme provides a useful and practical control methodology for the control of large flexible spacecraft.

Appendix: Comparison of Junkins' Control Law with the Present Law

In Refs. 1 and 2, Junkins et al. present a Lyapunov control law design for the maneuver and vibration suppression of flexible systems. The model considered was a rigid hub with flexible appendage beams undergoing a single axis maneuver, and each beam had a finite tip mass. For easy comparison, only one appendage and no tip masses are adopted in the following development of Junkins' control law. In terms of the notations of Ref. 1 the system equations of motion are

$$I_h \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = u(t) + (M_0 - S_0 l_0) \quad \text{(A1)}$$

$$-(M_0 - S_0 l_0) = \int_{l_0}^{l} \rho x \left(\frac{\partial^2 y}{\partial t^2} + x \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \right) \mathrm{d}x \quad \text{(A2)}$$

$$\rho \left(\frac{\partial^2 y}{\partial t^2} + x \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \right) + EI \frac{\partial^4 y}{\partial x^4} = 0 + \mathrm{HOT} \quad \text{(A3)}$$

where (M_0, S_0) denotes the bending moment and the shear force, at the root of the beam, respectively, and Eq. (A3) satisfies the same boundary conditions as in Eq. (6).

After substitution of Eq. (A2) into Eq. (A1) one can obtain

$$(I_h + I_b)\ddot{\theta} + \frac{\mathrm{d}}{\mathrm{d}t} H_f = u(t) \quad \text{(A4)}$$

where H_f is the flexible momentum of the beam.

The control law of Ref. 1 is taken as the following form:

$$u(t) = -\left[g_1\theta + g_2\dot{\theta} + g_3(M_0 - S_0 l_0) \right] \quad \text{(A5)}$$

After substitution of Eq. (A2) into Eq. (A5), the control law can be rewritten as

$$u(t) = -\left[g_1\theta + g_2\dot{\theta} + g_3\left(I_b\ddot{\theta} + \frac{\mathrm{d}}{\mathrm{d}t}H_f\right)\right] \tag{A6}$$

Notice that this control law is different from Eq. (7) in that the third term of Eq. (A6) involves the extra time rate of change of the rigid momentum of the beam.

To show the difference, we use the same Lyapunov function (8) as the test function. In this case the same manipulations as those in Sec. II lead to the following requirements for the control gains:

$$g_2 \geq 0 \tag{A7}$$

$$g_1 = \frac{a_2[I_h + (1+g_3)I_b]}{a_3 I_b + a_1(I_h + I_b)} \tag{A8}$$

$$g_3 = -1 + \frac{a_3 I_h}{a_1(I_h + I_b)} \tag{A9}$$

In this case, the derivative of the Lyapunov function

$$\dot{V} = -\frac{a_1(I_h + I_b)}{I_h}\dot{\theta}^2 \tag{A10}$$

From Eq. (A9) we obtain

$$g_3 \geq 1 \qquad \text{and without finite upper bound}$$

Also, from Eqs. (A9) and (A10) it is seen that, if a_1 is fixed and if a_3 is increased (i.e., the increase of the relative contribution of the flexible motion to the error energy Lyapunov function), the gain g_3 will increase; however, the $\dot{V}$ is unchanged. This means physically that increasing the control gain g_3, unlike increasing the control gain γ, will not automatically increase the suppression of the flexible motion. Therefore, it is concluded that the inclusion of the rigid momentum into the feedback control law is unnecessary and may corrupt the feedback information with possible degradation of the system performance.

Finally, it is easily seen from Eq. (A2) that the combination of the root shear and bending moment is equal to the time rate of change of the rigid momentum plus the flexible momentum of the beam. Therefore, the flexible momentum feedback term can be constructed by measuring the root shear force, root bending moment, and the angular acceleration of the hub.

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